Homework 1: Task 1

Consider the system

$$
\begin{aligned}
\frac{d}{dt}x &= -y + \varepsilon \cdot x \cdot \left(x^2 + y^2 - 1\right) \\
\frac{d}{dt}y &= x + \delta \cdot y^3 \cdot \left(x^2 + y^2 - 1\right)
\end{aligned}
$$

For which values of the constants ε and δ the periodic solution

$$x_\star(t) = cost, \qquad y_\star(t) = \sin t$$

is asymptotically orbitally stable?

Derive the answer analyzing

1. the first order approximation of the Poincare first return map;
2. the time derivative of the Lyapunov function candidate

$$V(x, y) = \frac{1}{2}\left(x^2 + y^2 - 1\right)^2$$

Homework 1: Task 2

The improved 0-order approximation of the stable cycle $x_*(\cdot)$ of

$$\ddot{x} + \omega^2 x = \mu(1 - x^2)\dot{x} \qquad \text{(Van der Pol equation)}$$

has the form

$$x_*(t) \approx a(t) \cos \psi(t) \quad + \mu x_{(1)}(t) + \mu^2 x_{(2)}(t) + \cdots + \mu^n x_{(k)}(t)$$

Here $a(\cdot) \geq 0$ and $\psi(\cdot)$ are functions defined by equations

$$\frac{d}{dt} a = \mu A_1(a) \quad + \mu^2 A_2(a) + \cdots + \mu^n A_n(a) \qquad (1)$$

$$\frac{d}{dt} \psi = \omega + \mu B_1(a) \quad + \mu^2 B_2(a) + \cdots + \mu^n B_n(a) \qquad (2)$$

with the functions $A_1(a) = \frac{a}{2}\left(1 - \frac{a^2}{4}\right)$ and $B_1(a) = 0$.

Check an asymptotic (in)stability of all equilibriums of (1).