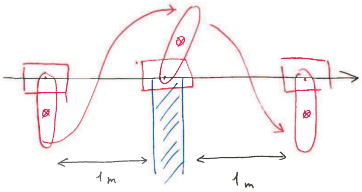
# Homework 3: Task 1

The dynamics of the cart-pendulum system are

$$\begin{aligned} 2 \cdot \ddot{x} + \cos\theta \cdot \ddot{\theta} - \sin\theta \cdot \dot{\theta}^2 &= f \\ \cos\theta \cdot \ddot{x} + \ddot{\theta} - g \cdot \sin\theta &= 0 \end{aligned}$$

where $x$ is a coordinate for representing a position of the cart; $\theta$ is an angle the pendulum makes with the vertical; and $f$ is an external force (control signal) that can be applied to the cart. The task is to find an external force (feedforward control signal) such that in response the pendulum of the system comes over a wall without collision.



# Homework 3: Task 2

Given a nominal circular motion $[x_c(t), y_c(t), \theta_c(t)]$

$$
\begin{aligned}
x_c(t) &= R \cdot \sin\theta_c(t) & y_c(t) &= -R \cdot \cos\theta_c(t) \\
\theta_c(t) &= \omega_c \cdot t + \theta_0 & u_c(t) &= 0
\end{aligned}
$$

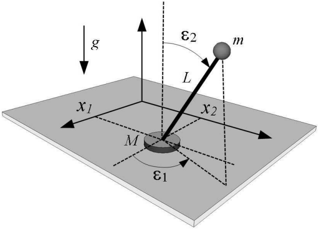
of the system

$$
\begin{aligned}
\ddot{x} &= -[\dot{y} \cdot \sin\theta + \dot{x} \cdot \cos\theta] \cdot \dot{\theta} \cdot \sin(\theta) \\
\ddot{y} &= [\dot{y} \cdot \sin\theta + \dot{x} \cdot \cos\theta] \cdot \dot{\theta} \cdot \cos(\theta) \\
J\ddot{\theta} &= u
\end{aligned}
$$

The task is to introduce as many as possible independent scalar functions $F(\cdot)$ of the state of the system that are zero on the nominal behavior

$$
F(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})\Big|_{\substack{x=x_c(t),\, y=y_c(t),\, \theta=\theta_c(t) \\ \dot{x}=\dot{x}_c(t),\, \dot{y}=\dot{y}_c(t),\, \dot{\theta}=\dot{\theta}_c(t)}} \equiv 0, \quad \forall\, t
$$

# Homework 3: Task 3




The system has four generalized coordinates: $(x_1, x_2)$ are variables for representing the position of the puck on a plane; $(\varepsilon_1, \varepsilon_2)$ are two angles (precession and nutation) for representing the status of the pendulum.

The dynamics of the system are

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{\varepsilon}_1}\right] - \frac{\partial \mathcal{L}}{\partial \varepsilon_1} = 0 \qquad \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{\varepsilon}_2}\right] - \frac{\partial \mathcal{L}}{\partial \varepsilon_2} = 0$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{x}_1}\right] - \frac{\partial \mathcal{L}}{\partial x_1} = F_1 \qquad \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{x}_2}\right] - \frac{\partial \mathcal{L}}{\partial x_2} = F_2$$

with $F_1$, $F_2$ being external forces acting on the puck.

The task: to find at least one behavior of the system when the puck is forced to move along of a circle of a radius $R$ while the pendulum stays above the horizontal for all the time.