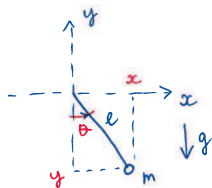


Homework 2: Task 1



Consider the point-mass system moving in the vertical plane (x, y) in presence of the gravity assuming that any of its motions is consistent with the constraint: $x(t)^2 + y(t)^2 - l^2 \equiv 0$.

The dynamics of the system written in excessive coordinates (x, y) are

$$\begin{aligned} m \cdot \ddot{x} &= \frac{m}{l^2} (y \cdot g - \dot{x}^2 - \dot{y}^2) \cdot x \\ m \cdot \ddot{y} &= \frac{m}{l^2} (y \cdot g - \dot{x}^2 - \dot{y}^2) \cdot y - m \cdot g \end{aligned} \quad (1)$$

and the dynamics written in generalized coordinate θ

$$\ddot{\theta} + \frac{g}{l} \cdot \sin \theta = 0 \quad (2)$$

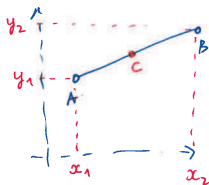
Check that a solution of (1) given by

$$x(t) = l \cdot \sin \theta(t), \quad y(t) = -l \cdot \cos \theta(t)$$

is determined by the system (2).

Homework 2: Task 2

Consider two point masses of $m = 1$ [kg] each connected by massless rod of length l and moving in the vertical plane.



Constraint No. 1: $(x_1(t) - x_2(t))^2 + (y_1(t) - y_2(t))^2 = l^2, \forall t$

Constraint No. 2: Assume that the velocity of the center of the rod – point C on the plot

$$\vec{v}_C = \left[\frac{1}{2}(\dot{x}_1 + \dot{x}_2); \frac{1}{2}(\dot{y}_1 + \dot{y}_2) \right]$$

always aligned with the rod written as the identity

$$(x_2(t) - x_1(t)) (\dot{y}_1(t) + \dot{y}_2(t)) - (y_2(t) - y_1(t)) (\dot{x}_1(t) + \dot{x}_2(t)) \equiv 0$$

Homework 2: Task 2

Assignments:

- Write the dynamics of the system
 - Integrate the dynamics, i.e. given initial conditions, find the corresponding solution of the system as a function of time
-

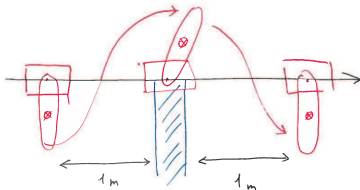
If necessary, use the (hand written) materials provided for solving the task 😊

Homework 2: Task 3

The dynamics of the cart-pendulum system are

$$\begin{aligned}2 \cdot \ddot{x} + \cos \theta \cdot \ddot{\theta} - \sin \theta \cdot \dot{\theta}^2 &= f \\ \cos \theta \cdot \ddot{x} + \ddot{\theta} - g \cdot \sin \theta &= 0\end{aligned}$$

where x is a coordinate for representing a position of the cart; θ is an angle the pendulum makes with the vertical; and f is an external force (control signal) that can be applied to the cart. The task is to find an external force (feedforward control signal) such that in response the pendulum of the system comes over a wall without collision.



Homework 2: Task 4

Given a nominal circular motion $[x_c(t), y_c(t), \theta_c(t)]$

$$\begin{aligned}x_c(t) &= R \cdot \sin \theta_c(t) & y_c(t) &= -R \cdot \cos \theta_c(t) \\ \theta_c(t) &= \omega_c \cdot t + \theta_0 & u_c(t) &= 0\end{aligned}$$

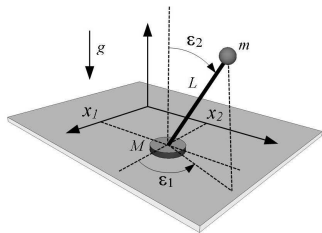
of the system (a coin rolling on a table, see Lecture 4 slides)

$$\begin{aligned}\ddot{x} &= -[\dot{y} \cdot \sin \theta + \dot{x} \cdot \cos \theta] \cdot \dot{\theta} \cdot \sin(\theta) \\ \ddot{y} &= [\dot{y} \cdot \sin \theta + \dot{x} \cdot \cos \theta] \cdot \dot{\theta} \cdot \cos(\theta) \\ J\ddot{\theta} &= u\end{aligned}$$

The task is to introduce as many as possible independent scalar functions $F(\cdot)$ of the state of the system that are zero on the nominal behavior

$$F(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta}) \Big|_{\substack{x=x_c(t), y=y_c(t), \theta=\theta_c(t) \\ \dot{x}=\dot{x}_c(t), \dot{y}=\dot{y}_c(t), \dot{\theta}=\dot{\theta}_c(t)}} \equiv 0, \quad \forall t$$

Homework 2: Task 5



The system has four generalized coordinates: (x_1, x_2) are variables for representing the position of the puck on a plane; $(\varepsilon_1, \varepsilon_2)$ are two angles (precession and nutation) for representing the status of the pendulum.

The dynamics of the system are

$$\begin{aligned} \frac{d}{dt} \left[\frac{\partial \mathcal{L}}{\partial \dot{\varepsilon}_1} \right] - \frac{\partial \mathcal{L}}{\partial \varepsilon_1} &= 0 & \frac{d}{dt} \left[\frac{\partial \mathcal{L}}{\partial \dot{\varepsilon}_2} \right] - \frac{\partial \mathcal{L}}{\partial \varepsilon_2} &= 0 \\ \frac{d}{dt} \left[\frac{\partial \mathcal{L}}{\partial \dot{x}_1} \right] - \frac{\partial \mathcal{L}}{\partial x_1} &= F_1 & \frac{d}{dt} \left[\frac{\partial \mathcal{L}}{\partial \dot{x}_2} \right] - \frac{\partial \mathcal{L}}{\partial x_2} &= F_2 \end{aligned}$$

with F_1, F_2 being external forces acting on the puck.

The task: to find at least one behavior of the system when the puck is forced to move along of a circle of a radius R while the pendulum stays above the horizontal for all the time.