Homework 4: Task 1

The dynamics of the cart-pendulum system are

$$2 \cdot \ddot{x} + \cos\theta \cdot \ddot{\theta} - \sin\theta \cdot \dot{\theta}^2 = f, \quad \cos\theta \cdot \ddot{x} + \ddot{\theta} - g \cdot \sin\theta = 0,$$

where x is a position of the cart; θ is an angle that the pendulum makes with the vertical; f is a control signal (force) applied to the cart. The task is to search for its forced periodic motion in a vicinity of the equilibrium

$$x_e = 0, \qquad \theta_e = 0.$$

The assignment is assumed to be solved based on analysis of the nested representation of system's motions, where

1. the position of the cart $x(\cdot)$ is chosen as a motion generator; and
2. the angle $\theta(\cdot)$ on the motion is computed as: $\theta = x + L \cdot \sin x$.

In solving the task, you are to analyze the dynamics of the motion generator and to compute the range of the constant parameter L, for which such system has a centre in a vicinity of the equilibrium $x_e = 0$.

Homework 4: Task 2

Given a nominal T-periodic motion $[x_*(t), \theta_*(t)]$ of the cart-pendulum found in Task 1, find a feedback transformation

$$f \to v$$

that brings the dynamics of the system into the format

$$\begin{aligned} \alpha(x)\ddot{x} + \beta(x)\dot{x}^2 + \gamma(x) &= g_y(\cdot)y + g_{\dot{y}}(\cdot)\dot{y} + g_v(\cdot)v \\ \ddot{y} &= v \end{aligned}$$

Here the variable $y(\cdot)$ is defined as a mismatch of the kinematic relation

$$y := \theta - \phi(x) = \theta - [x + L \cdot \sin x]$$

Compute the linearization of the transverse dynamics of the system in a vicinity of the nominal T-periodic solution using the transverse coordinates $[I(\cdot);\ y(\cdot);\ \dot{y}(\cdot)]$ as proposed in the lecture slides.