

Exercise 1

Consider the system

$$\begin{aligned}\frac{d}{dt}x &= -y + \varepsilon \cdot x \cdot (x^2 + y^2 - 1) \\ \frac{d}{dt}y &= x + \delta \cdot y^3 \cdot (x^2 + y^2 - 1)\end{aligned}$$

For which values of the constants ε and δ the periodic solution

$$x_*(t) = \cos t, \quad y_*(t) = \sin t$$

is asymptotically orbitally stable?

Derive the answer analyzing

1. the first order approximation of the Poincare first return map;
2. the time derivative of the Lyapunov function candidate

$$V(x, y) = \frac{1}{2} (x^2 + y^2 - 1)^2$$