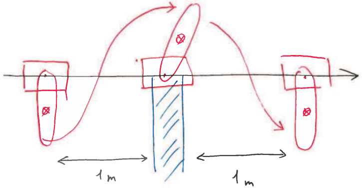
Exercise 3: Task 1

The dynamics of the cart-pendulum system are

$$\begin{aligned} 2 \cdot \ddot{x} + \cos\theta \cdot \ddot{\theta} - \sin\theta \cdot \dot{\theta}^2 &= f \\ \cos\theta \cdot \ddot{x} + \ddot{\theta} - g \cdot \sin\theta &= 0 \end{aligned}$$

where x is a coordinate for representing a position of the cart; θ is an angle the pendulum makes with the vertical; and f is an external force (control signal) that can be applied to the cart. The task is to complete the lecture's arguments and to find an external force $f(\cdot)$ such that in response the pendulum of the system comes over a wall without collision.



Exercise 3: Task 2

Given a nominal circular motion $[x_c(t), y_c(t), \theta_c(t)]$

$$
\begin{aligned}
x_c(t) &= R \cdot \sin\theta_c(t) & \qquad y_c(t) &= -R \cdot \cos\theta_c(t) \\
\theta_c(t) &= \omega_c \cdot t + \theta_0 & \qquad {\color{red}u_c(t)} &\equiv 0
\end{aligned}
$$

of the system (a coin rolling on a table, see Lecture 5 slides)

$$
\begin{aligned}
\ddot{x} &= -[\dot{y} \cdot \sin\theta + \dot{x} \cdot \cos\theta] \cdot \dot{\theta} \cdot \sin(\theta) \\
\ddot{y} &= [\dot{y} \cdot \sin\theta + \dot{x} \cdot \cos\theta] \cdot \dot{\theta} \cdot \cos(\theta) \\
J\ddot{\theta} &= {\color{red}u}
\end{aligned}
$$

The task is to introduce as many as possible independent scalar functions $F(\cdot)$ of the state of the system that are zero on the nominal behavior

$$
F(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})\Big|_{\substack{x=x_c(t),\, y=y_c(t),\, \theta=\theta_c(t) \\ \dot{x}=\dot{x}_c(t),\, \dot{y}=\dot{y}_c(t),\, \dot{\theta}=\dot{\theta}_c(t)}} \equiv 0, \quad \forall\, t
$$