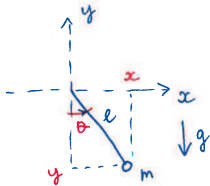


## Homework 3: Task 1



Consider the point-mass system moving in the vertical plane  $(x, y)$  in presence of the gravity assuming that any of its motions is consistent with the constraint:  $x(t)^2 + y(t)^2 - l^2 \equiv 0$ .

The dynamics of the system written in excessive coordinates  $(x, y)$  are

$$\ddot{x} = \frac{1}{l^2} \cdot (y \cdot g - \dot{x}^2 - \dot{y}^2) \cdot x, \quad \ddot{y} = \frac{1}{l^2} \cdot (y \cdot g - \dot{x}^2 - \dot{y}^2) \cdot y - g \quad (1)$$

and the dynamics written in generalized coordinate  $\theta$  are

$$\ddot{\theta} + \frac{g}{l} \cdot \sin \theta = 0 \quad (2)$$

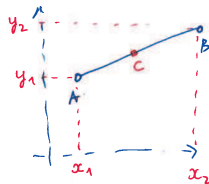
Given a solution  $\theta(t) = \theta(t, \theta_0, \dot{\theta}_0)$  of the system (2), check that the functions defined as

$$x(t) = l \cdot \sin \theta(t), \quad y(t) = -l \cdot \cos \theta(t),$$

by necessity, describe one of solutions of the system (1).

## Homework 3: Task 2

Consider two point masses of  $m = 1$  [kg] each connected by massless rod of length  $l$  and moving in the vertical plane subject to the constraints.



Constraint No. 1:  $\left(x_1(t) - x_2(t)\right)^2 + \left(y_1(t) - y_2(t)\right)^2 = l^2, \forall t$

Constraint No. 2: Assume that the velocity of the center of the rod – point **C** on the plot

$$\vec{v}_C = \left[\frac{1}{2}(\dot{x}_1 + \dot{x}_2); \frac{1}{2}(\dot{y}_1 + \dot{y}_2)\right]$$

always aligned with the rod, i.e. the following relation holds  $\forall t$

$$\left(x_2(t) - x_1(t)\right) \left(\dot{y}_1(t) + \dot{y}_2(t)\right) - \left(y_2(t) - y_1(t)\right) \left(\dot{x}_1(t) + \dot{x}_2(t)\right) \equiv 0$$

## Homework 3: Task 2 (cont'd)

Assignments:

- Write the dynamics of the system
- Integrate the dynamics, i.e. given initial conditions, find the corresponding solution of the system as a function of time

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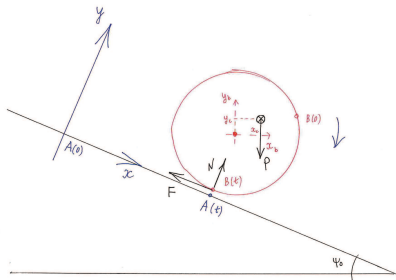
If necessary, use the (hand written) materials provided for solving the task 😊

## Homework 3: Task 3

Consider a disc of mass  $m_d$ , radius  $R_d$ , moment of inertia  $J_z$  and location of the center of mass

$$\vec{r}_C = [x_C; y_C] \neq [0; 0],$$

which rolls down the slope having the angle  $\psi_0$  with the horizontal.



Assignments/Questions:

- Write the dynamics of the system.
- Can the system has a behavior, where the disc slides for some interval of time and rolls without sliding for another?

## Homework 3: Task 4

Consider a disc of mass  $m_d$ , radius  $R_d$ , moment of inertia  $J_z$  and with location of the center of mass at

$$\vec{r}_C = [x_C; y_C] = [0; 0],$$

which rolls on the prism possessing the angle  $\psi_0$  with horizontal. In turn, the prism of the mass  $m_p$  with coordinates of the center of mass at  $[x_p; y_p]$  can slide without friction along the horizontal.

Assignments/Questions:

- Write the dynamics of the system.
- Can the system has a behavior, where the disc slides for some interval of time and rolls without sliding for another?

