

Homework 5

Given a nominal circular motion $[x_c(t), y_c(t), \theta_c(t)]$

$$\begin{aligned}x_c(t) &= R \cdot \sin \theta_c(t) & y_c(t) &= -R \cdot \cos \theta_c(t) \\ \theta_c(t) &= \omega_c \cdot t + \theta_0 & \textcolor{red}{u}_c(t) &\equiv 0\end{aligned}$$

of the system (a coin rolling on a table, see Lecture 5 slides)

$$\begin{aligned}\ddot{x} &= -[\dot{y} \cdot \sin \theta + \dot{x} \cdot \cos \theta] \cdot \dot{\theta} \cdot \sin(\theta) \\ \ddot{y} &= [\dot{y} \cdot \sin \theta + \dot{x} \cdot \cos \theta] \cdot \dot{\theta} \cdot \cos(\theta) \\ J\ddot{\theta} &= \textcolor{red}{u}\end{aligned}$$

The task is to introduce as many as possible **independent** scalar functions $F(\cdot)$ of the state of the system that are zero on the nominal behavior

$$F(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta}) \Big|_{\substack{x=x_c(t), y=y_c(t), \theta=\theta_c(t) \\ \dot{x}=\dot{x}_c(t), \dot{y}=\dot{y}_c(t), \dot{\theta}=\dot{\theta}_c(t)}} \equiv 0, \quad \forall t$$