Homework 1

Consider a family of systems parametrized by constants ε and δ

$$\frac{d}{dt}x = -y + \varepsilon \cdot x \cdot \left(x^2 + y^2 - 1\right), \quad \frac{d}{dt}y = x + \delta \cdot y^3 \cdot \left(x^2 + y^2 - 1\right)$$

Each of them has the following periodic solution

$$x_\star(t) = \cos t, \qquad y_\star(t) = \sin t \tag{1}$$

Determine ranges of the parameters ε and δ, for which the periodic solution (1) is asymptotically orbitally stable.

Derive the answer analyzing

1. the first order approximation of the Poincare first return map;
2. the time derivative of the Lyapunov function candidate

$$V(x, y) = \tfrac{1}{2}\left(x^2 + y^2 - 1\right)^2$$