Homework 2

Consider the state space form $\dot{x} = f(x)$ of the closed loop system

$$a \cdot \ddot{\theta} + c \cdot \sin\theta = u, \qquad u = -\varphi(\dot{\theta}) \cdot \Big[E\big(\theta, \dot{\theta}\big) - E_0\Big] \tag{1}$$

with $\{x_1 = \theta;\ x_2 = \dot{\theta}\}$ written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{c}{a} \cdot \sin x_1 - \frac{\varphi(x_2)}{a} \cdot \left[\frac{a}{2} \cdot x_2^2 + c \cdot (1 - \cos x_1) - E_0\right] \end{bmatrix}$$

Consider the linear system

$$\frac{d}{dt} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathcal{A}(t) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{2}$$

defined for the solution $x_1^0(t) = \theta_0(t)$, $x_2^0(t) = \dot{\theta}_0(t)$ of the closed loop nonlinear system of the given energy value

$$\frac{a}{2} \cdot \left[x_2^0(0)\right]^2 + c \cdot (1 - \cos x_1^0(0)) = E_0$$

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and obtained with help of Leonov's lemma as

$$\mathcal{A}(t) = \left\{ \frac{\partial f(x)}{\partial x} - \frac{f(x) f(x)^T}{|f(x)|^2} \left(\frac{\partial f(x)}{\partial x} + \left[\frac{\partial f(x)}{\partial x} \right]^T \right) \right\} \Bigg|_{x = x^0(t)}$$

Task 1: Derive coefficients of the matrix-function $\mathcal{A}(t)$ of the transverse linearization (2).

Task 2: Find solutions $v^{\perp}(\cdot)$ of the transverse linearization (2) that can be used for approximating the transverse dynamics of the nonlinear system (1) in a vicinity of the nominal solution $x^0(t)$.