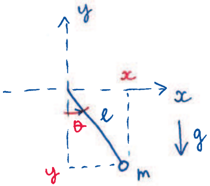
Homework 3: Task 1



Consider the point-mass system moving in the vertical plane (x, y) in presence of the gravity assuming that any of its motions is consistent with the constraint:
$x(t)^2 + y(t)^2 - l^2 \equiv 0$.

The dynamics of the system written in excessive coordinates (x, y) are

$$\ddot{x} = \tfrac{1}{l^2} \cdot (y \cdot g - \dot{x}^2 - \dot{y}^2) \cdot x, \qquad \ddot{y} = \tfrac{1}{l^2} \cdot (y \cdot g - \dot{x}^2 - \dot{y}^2) \cdot y - g \tag{1}$$

and the dynamics written in generalized coordinate θ are

$$\ddot{\theta} + \tfrac{g}{l} \cdot \sin\theta = 0 \tag{2}$$

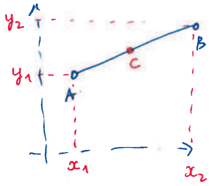
Given a solution $\theta(t) = \theta(t, \theta_0, \dot{\theta}_0)$ of the system (2), check that the functions defined as

$$x(t) = l \cdot \sin\theta(t), \quad y(t) = -l \cdot \cos\theta(t),$$

by necessity, describe one of solutions of the system (1).

Homework 3: Task 2

Consider two point masses of $m = 1$ [kg] each connected by massless rod of length l [m] and moving in the vertical plane subject to the constraints:



Constraint No. 1: $\Big(x_1(t) - x_2(t)\Big)^2 + \Big(y_1(t) - y_2(t)\Big)^2 = l^2, \ \forall\, t$

Constraint No. 2: Assume that the velocity of the center of the rod – point C on the plot

$$\vec{v}_C = \left[\tfrac{1}{2}(\dot{x}_1 + \dot{x}_2);\ \tfrac{1}{2}(\dot{y}_1 + \dot{y}_2)\right]$$

always aligned with the rod, i.e. the following relation holds $\forall\, t$

$$\Big(x_2(t) - x_1(t)\Big)\Big(\dot{y}_1(t) + \dot{y}_2(t)\Big) - \Big(y_2(t) - y_1(t)\Big)\Big(\dot{x}_1(t) + \dot{x}_2(t)\Big) \equiv 0$$

Homework 3: Task 2 (cont'd)

Assignments:

- Write the dynamics of the system
- Integrate the dynamics, i.e. given initial conditions, find the corresponding solution of the system as a function of time

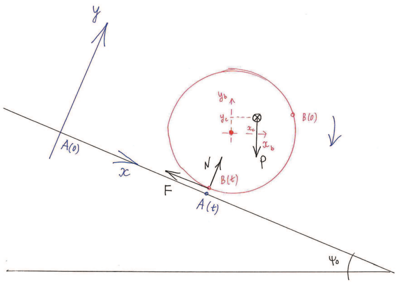
---

If necessary, use the (hand written) materials provided for solving the task ☺

Given a disc of the mass m_d, radius R_d, moment of inertia J_z and with the location of the center of mass

$$\vec{r}_C = [x_C; y_C] \neq [x_S; y_S],$$

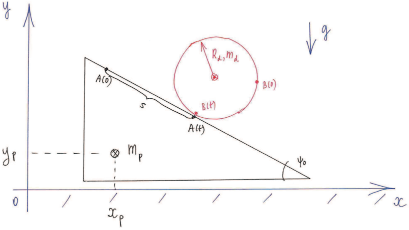
being different from the centre of the disc symmetry, suppose the disc rolls on the slope that is of angle ψ_0 with the horizontal.



Assignments/Questions:

- Write the dynamics of the system.
- Can the system has a behavior, where the disc first rolls and slides for some interval of time and then rolls on the slope without slipping for another interval of time?

Consider a disc of mass m_d, radius R_d, moment of iner of mass at



$$\color{red}{\vec{r}_C} = [\color{red}{x_C}; \color{red}{y_C}] = [x_S; y_S],$$

which rolls on the prism possessing the angle ψ_0 with horizontal. In turn, the prism of the mass m_p with coordinates of the center of mass at $[x_p; y_p]$ can slide without friction along the horizontal.

Assignments/Questions:

- Write the dynamics of the system.
- Can the system has a behavior, where the disc first rolls and slides for some interval of time and then rolls on the prism without slipping for another interval of time?