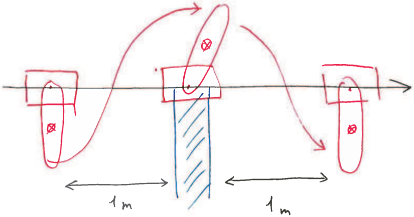
Homework 4: Task 1

The dynamics of the cart-pendulum system are

$$
\begin{aligned}
2 \cdot \ddot{x} + \cos\theta \cdot \ddot{\theta} - \sin\theta \cdot \dot{\theta}^2 &= f \\
\cos\theta \cdot \ddot{x} + \ddot{\theta} - g \cdot \sin\theta &= 0
\end{aligned}
$$

where x is a position of the cart on the horizontal; θ is an angle that the pendulum makes with the vertical; f is an external force (control signal) applied to the cart. The task is to find an external force (feedforward control signal) following the arguments of the lecture such that in response to the input the pendulum of the system comes over a wall without collision



Homework 4: Task 2

Coordinates q of a planar mobile robot are the position of its center of mass $[x, y] \in \mathbb{R}^2$ and its heading angle $\theta \in \mathbb{S}^1$. Suppose the dynamics of the robot are subject to an ideal constraint

$$f(q, \dot{q}) = \dot{y} \cdot \cos\theta - \dot{x} \cdot \sin\theta \tag{1}$$

and, therefore, defined by the Newton-Euler equations

$$m\ddot{x} = R_x^c, \qquad m\ddot{y} = R_y^c, \qquad J\ddot{\theta} = u + M_z^c,$$

where m, J are the mass and the moment of inertia of the robot; $R^c = [R_x^c; R_y^c]$ is the constraint force due to (1) and M_z^c is the torque generated by the constraint force R^c; u is a control signal (external torque) that can be applied to modify the system dynamics.

The task is to verify that the constraint $f(q, \dot{q}) = 0$ in Eqn. (1) cannot be integrated and re-written as holonomic $g(q) = 0$ for some smooth function of coordinates $g(\cdot)$ irrespective of choice of the control variable u.

Homework 4: Task 3

Given constants ω_c, ψ_0, consider the nominal forced motion

$$x_c(t) = R_d \cdot \cos\psi_\heartsuit(t), \quad y_c(t) = R_d \cdot \sin\psi_\heartsuit(t), \quad \theta_c(t) = \psi_\heartsuit(t) + \frac{\pi}{2}, \quad \psi_\heartsuit(t) = \omega_c \cdot t + \psi_0$$

of the system

$$\begin{aligned} \ddot{x} &= -\left[\dot{y} \cdot \sin\theta + \dot{x} \cdot \cos\theta\right] \cdot \dot{\theta} \cdot \sin(\theta) \\ \ddot{y} &= \left[\dot{y} \cdot \sin\theta + \dot{x} \cdot \cos\theta\right] \cdot \dot{\theta} \cdot \cos(\theta) \\ J\ddot{\theta} &= u + M_z^c \end{aligned}$$

Questions and assignments:

1. How many transverse coordinates are required for defining the transverse dynamics for the nominal motion? Propose candidates for transverse coordinates for the nominal motion.
2. Does the system have integrals of motion? How many?
3. Derive the linearization of the transverse coordinates in a vicinity of the nominal motion.